Mechanical Characterization of Structured Sheet Materials - Supplemental Material

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1 DIRECTION-DEPENDENT ELASTICITY MEASURES

We use three elasticity parameters, the Young's modulus, Poisson's ratio, and bending stiffness, as a way to visualize the direction-dependent material behavior of structured sheet materials in an intuitive way. While the formulas to compute these properties from the stiffness tensors are well-known [Böhlke and Brüggemann 2001], we think that a more thorough explanation of their derivation is important for the understanding of direction-dependent material behavior.

1.1 Membrane

We characterize the membrane behavior of a material using the Young's modulus and Poisson's ratio. Intuitively, these properties describe how much the material resists stretch along a direction, and how much the material contracts perpendicular to this direction. Both of these properties are measured in a *uniaxial stress* configuration, where the material is stretched along a given direction, and any stress perpendicular to this direction is resolved by deformation.

While the in-plane material behavior is most often described using the stiffness tensor $\mathbb{C}$, which maps strains to stresses, the case of uniaxial stress is more easily covered by the *compliance tensor* $\mathbb{S} = \mathbb{C}^{-1}$, its symmetric inverse, mapping stresses to strains.

Young's modulus. We can define a *uniaxial unit stress* along a given direction $\mathbf{d}$ using a simple outer product, $\boldsymbol{\sigma}^{\mathbf{d}} = \mathbf{d}\mathbf{d}^T$. Applying this stress to the compliance tensor results in the strain $\boldsymbol{\epsilon}^{\mathbf{d}} = \mathbb{S} : \boldsymbol{\sigma}^{\mathbf{d}}$ that is induced by this unit stress. From this strain tensor, we want to extract the deformation along the direction $\mathbf{d}$, which we get by again applying the tensor $\mathbf{d}\mathbf{d}^T$ to the strain, resulting in the expression $(\mathbf{d}\mathbf{d}^T) : \boldsymbol{\epsilon}^{\mathbf{d}}$. The ratio between the applied stress and the induced deformation then defines the Young's modulus, and since we used the unit stress, we arrive at the formula

$$E(\mathbf{d}) = \frac{1}{(\mathbf{d}\mathbf{d}^T) : \mathbb{S} : (\mathbf{d}\mathbf{d}^T)} \; . \tag{1}$$

Poisson's ratio. The computation of the Poisson's ratio is based on the same strain response $\boldsymbol{\epsilon}^{\mathbf{d}}$ to the uniaxial unit stress $\boldsymbol{\sigma}^{\mathbf{d}}$, but we now compare two different deformations instead of a deformation and stress. Additionally to the deformation along the direction $\mathbf{d}$, we therefore also need to extract the deformation along the direction $\mathbf{n}$ that is perpendicular to $\mathbf{d}$, which we get by applying the tensor $\mathbf{n}\mathbf{n}^T$ to $\boldsymbol{\epsilon}^{\mathbf{d}}$. The negative ratio between these two deformations defines the Poisson's ratio, resulting in the expression

$$\nu(\mathbf{d}) = -\frac{(\mathbf{d}\mathbf{d}^T) : \mathbb{S} : (\mathbf{n}\mathbf{n}^T)}{(\mathbf{d}\mathbf{d}^T) : \mathbb{S} : (\mathbf{d}\mathbf{d}^T)}. \tag{2}$$

1.2 Bending

We compute the bending properties based on a simpler approach. Instead of applying a uniaxial unit moment, we measure the bending stiffness on a deformation with purely cylindrical curvature directly on the bending stiffness tensor $\mathbb{B}$. Given a direction $\mathbf{d}$, we apply the unit curvature tensor $\boldsymbol{\kappa}^{\mathbf{d}} = \mathbf{d}\mathbf{d}^T$ to $\mathbb{B}$, which results in the bending moment $\mathbf{M}^{\mathbf{d}} = \mathbb{B} : \boldsymbol{\kappa}^{\mathbf{d}}$. From this bending moment, we can, similar to the membrane case, extract the directional bending moment by again applying the tensor $\mathbf{d}\mathbf{d}^T$ to it. This then gives us the formula to compute the bending stiffness as

$$b(\mathbf{d}) = (\mathbf{d}\mathbf{d}^T) : \mathbb{B} : (\mathbf{d}\mathbf{d}^T). \tag{3}$$

2 TENSILE TEST RESULTS

We present the full plots of our tensile test results in Figure **??**, Figure **??**, and Figure 1. For the anisotropic structures 5, 6, 7, and 8, we performed additional measurements on a rotated sample to capture the direction dependence of the tensile strength.

3 BENDING TEST RESULTS

We present the full plots of our bending test results in Figure **??** and Figure 2. For structure 7 and 8, we performed additional measurements on a rotated sample to investigate their anisotropy.

REFERENCES

Thomas Böhlke and C. Brüggemann. 2001. Graphical representation of the generalized Hooke's law. *Technische Mechanik* 21, 2, 145–158.

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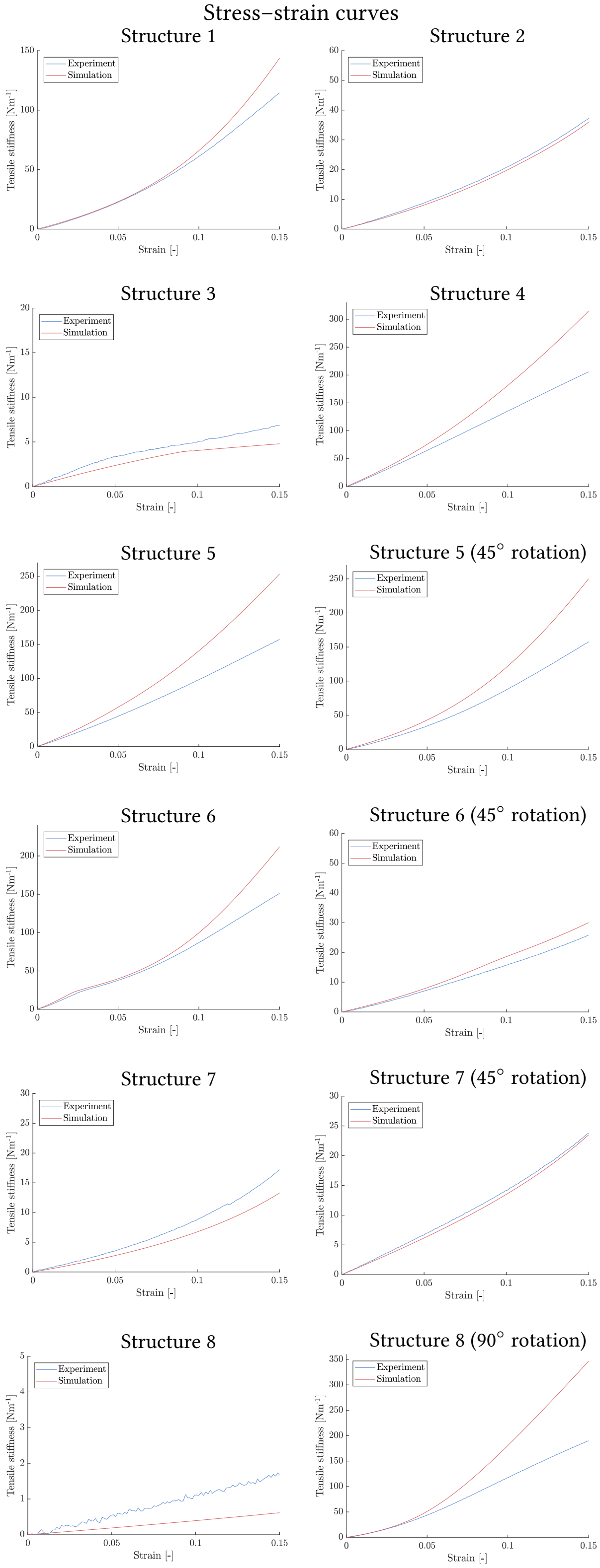

Fig. 1. Tensile test results and comparison to our simulation.

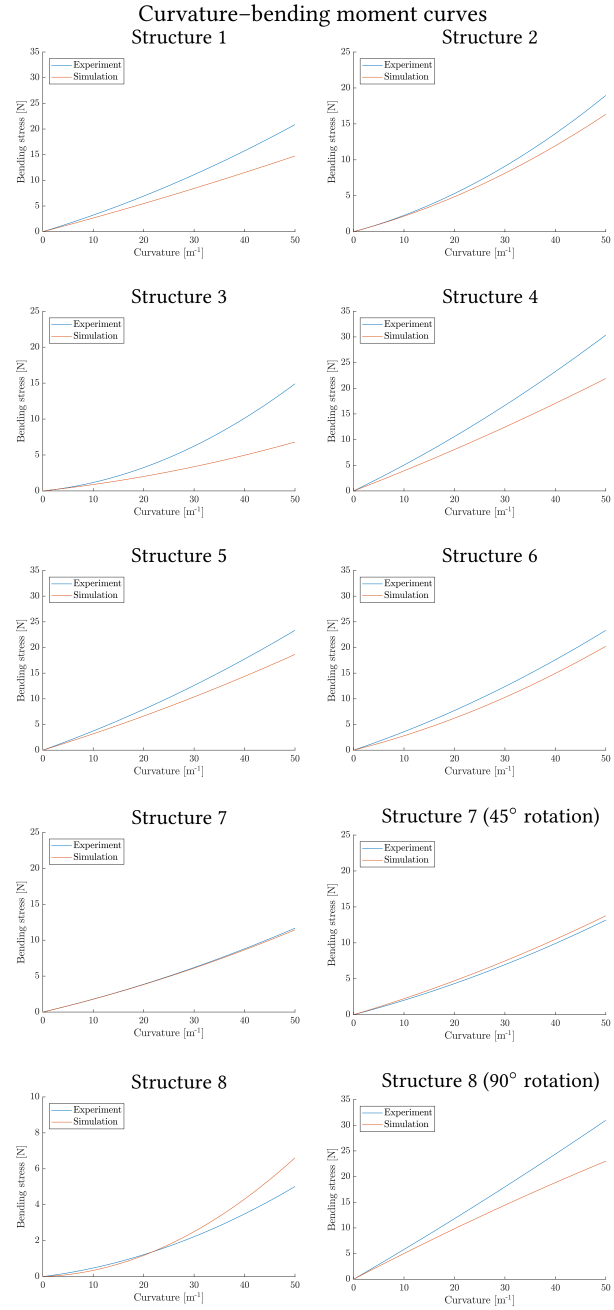

Fig. 2. Bending test results and comparison to our simulation.